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On the infrared sensitivity of the longitudinal cross section in e^+e^- annihilation[*]

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We have calculated the contributions proportional to $\beta_0^n \alpha_s^{n+1}$ to the longitudinal fragmentation function in e^+e^- annihilation to all orders of perturbation theory. We use this result to estimate higher-order perturbative corrections and nonperturbative corrections to the longitudinal cross section σ_L and discuss the prospects of determining α_s from σ_L. The structure of infrared renormalons in the perturbative expansion suggests that the longitudinal cross section for hadron production with fixed momentum fraction x receives nonperturbative contributions of order $1/(x^2 Q^2)$, whereas the total cross section has a larger, $1/Q$ correction. This correction arises from very large longitudinal distances and is related to the behaviour of the Borel integral for the cross section with fixed x at large values of the Borel parameter.

1. Introduction

The ALEPH [1] and OPAL [2] collaborations have measured the dependence of single-particle inclusive cross sections in e^+e^- annihilation on the scattering angle θ between the observed hadron h and the incoming electron beam. The angular dependence discriminates between contributions from transversely and longitudinally polarized virtual bosons, and from Z^0-photon interference [3]

$$\frac{d^2\sigma^h}{dx\, d\cos\theta}(e^+e^- \to hX) =$$
$$= \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_T^h}{dx}(x,Q^2) + \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L^h}{dx}(x,Q^2)$$
$$+ \frac{3}{4}\cos\theta\,\frac{d\sigma_A^h}{dx}(x,Q^2). \qquad (1)$$

In the following, dropping the superscript 'h' implies summation over all hadrons h.

In this paper we concentrate on the longitudinal cross section. It is given as a convolution of a parton fragmentation function D_p^h $(p = q, \bar{q}, g)$ with a partonic cross section $d\hat{\sigma}_L^p/dx$

$$\frac{d\sigma_L^h}{dx}(x,Q^2) = \sum_p \int_x^1 \frac{dz}{z}\frac{d\hat{\sigma}_L^p}{dz}(z)D_p^h(x/z,Q^2). \quad (2)$$

The perturbative expansion of the longitudinal parton cross section starts at order α_s.

Summed over all hadrons, the fragmentation functions satisfy the energy conservation sum rule $\sum_h \int_0^1 dx\, x D_p^h(x,Q^2) = 1$. Consequently, the integrated longitudinal cross section is an infrared (IR) safe quantity which is calculable in perturbation theory

$$\sigma_L \equiv \sum_h \frac{1}{2}\int_0^1 dx\, x\frac{d\sigma_L^h}{dx} = \sum_p \frac{1}{2}\int_0^1 dx\, x\frac{d\hat{\sigma}_L^p}{dx}$$
$$= \sigma_0\left[\frac{\alpha_s}{\pi} + (14.583 - 1.028 N_f)\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots\right]. (3)$$

Here σ_0 is the Born total e^+e^- annihilation cross section, $\alpha_s \equiv \alpha_s(Q)$ and N_f is the number of active fermion flavours. The next-to-leading order contribution has been obtained in [4]. OPAL [2] has measured σ_L the Z_0 peak:

$$\sigma_L/\sigma_{\text{tot}}(M_Z^2) = 0.057 \pm 0.005 . \qquad (4)$$

One of the main motivations for the present study is to investigate whether measurements of the total longitudinal cross sections can yield a precise determination of the strong coupling. This requires that we control higher-order perturbative corrections and nonperturbative effects, both of which are expected to be much larger for σ_L than for the total cross section σ_{tot}. We address both types of corrections in this report, by studying the structure of IR renormalons, a certain class of higher-order perturbative corrections, for the longitudinal cross section.

The study of nonperturbative effects in fragmentation functions is an interesting topic in its own right [5]. The light-cone expansions for fragmentation functions and for structure functions in DIS are similar [6], and suggest that nonperturbative effects in both cases are of order $1/Q^2$ and can be described in terms of multi-parton distributions. In contrast to DIS, however, the relevant operator structures for fragmentation are essentially nonlocal and cannot be expanded at small distances. Hence the usual operator product expansion does not apply and the status of the light-cone expansion is less established. Hadronization models generically introduce nonperturbative corrections of order $1/Q$, while current data on scaling violations in fragmentation do not distinguish between $1/Q$ or $1/Q^2$ behaviour. A nonperturbative correction of order $1/Q$ to the total longitudinal cross section was suggested in [7] as a consequence of phase-space reduction in the one-loop diagram calculated with a massive gluon. We address these apparently conflicting statements below.

While this work was in writing, Dasgupta and Webber [8] have addressed a similar set of questions with closely related methods.
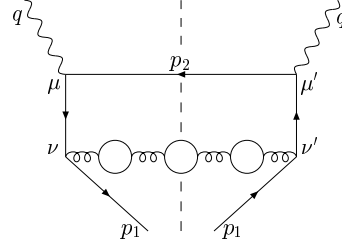

Figure 1. The 'primary' quark contribution to σ_L. Sum over all possible insertions of the bubble chain is understood.

2. General formalism

There is suggestive evidence from exact low-order results that β_0 is a large parameter and that keeping corrections of order $(\beta_0 \alpha_s)^n$ in higher orders resums important contributions. Moreover, the infrared renormalons encoded in the corresponding series can elucidate the power-behaviour of nonperturbative corrections and, perhaps, even their x-dependence, as in the case of $d\sigma_L/dx$. The $(\beta_0 \alpha_s)^n$ corrections can be traced by inserting a chain of fermion loops into the gluon propagator, and by restoring the full QCD β-function coefficient $\beta_0 = -1/(4\pi)[11 - 2/3N_f]$ from the dependence on N_f. For σ_L we obtain two contributions, according to whether the registered parton comes from the primary vertex or a fermion loop. The corresponding diagrams are shown in Fig. 1 and Fig. 2, for the contributions of the 'primary' and 'secondary' quarks, respectively. Note that the secondary quark contribution reduces to the gluon contribution at lowest order in α_s.

The evaluation of the two classes of diagrams, for an arbitrary number of internal fermion loops, and for their sum, is relatively straightforward by means of the dispersion technique developed in [10–12], in terms of the distribution function over the invariant mass k^2 of the bubble chain:

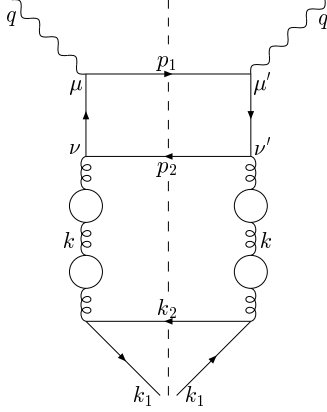$$\frac{d\hat{\sigma}_L^{[\text{p,s}]}}{dx}(x, \xi = k^2/Q^2) \equiv$$

Figure 2. The 'secondary' quark contribution to σ_L. Sum over all possible insertions of the bubble chain is understood.

$$
\begin{aligned}
\equiv \quad & \frac{8\pi}{N_c q^2} \int d\,\mathrm{Lips}[p_1, p_2, k]\, \frac{1}{k^2} \mathcal{M}_{\mu\nu} \mathcal{M}^*_{\mu'\nu'} \\
\times\ & \int d\,\mathrm{Lips}[k_1, k_2]\, \frac{1}{2} \mathrm{Tr}[\gamma_\nu\, \not{k}_1 \gamma_{\nu'}\, \not{k}_2] \\
\times\ & \begin{cases} \frac{p_{1,\mu} p_{1,\mu'}}{2|\vec{p}_1|^2} \delta\left(x - \frac{2p_1 q}{q^2}\right) & \text{'primary'} \\[2mm] \frac{k_{1,\mu} k_{1,\mu'}}{2|\vec{k}_1|^2} \delta\left(x - \frac{2k_1 q}{q^2}\right) & \text{'secondary'} \end{cases}
\end{aligned} \tag{5}
$$

The notation for momenta and Lorenz indices corresponds to Fig. 2. We denote by $[p]$ ($[s]$) the contribution of the 'primary' ('secondary') quark, while $\int d\,\mathrm{Lips}[\ldots]$ are Lorentz-invariant phase space integrals with the momentum conservation δ-function included, and $\mathcal{M}_{\mu\nu} \mathcal{M}^*_{\mu'\nu'}$ is the matrix element for the primary $q\bar{q}g$ amplitude squared.

Note that in the case of the 'primary' quark contribution the phase space integral over k_1, k_2 is proportional to k^2, so that the result takes the form of the one-loop diagram calculated with a gluon of mass k^2. This equivalence does not hold for the registered 'secondary' quark because of the nontrivial longitudinal projector. Because of this inequivalence, the restoration of β_0 from fermion loops is not unambiguous and the relation of fermion loop chains with running coupling effects is partially lost. In practice, we have found that the numerical differences are small, so that a detailed discussion is deferred to [5]. The analytic expressions for the primary and secondary quark contribution are rather lengthy and will also be given there.

Given the invariant mass distributions (in $\xi = k^2/Q^2$), finite order results are obtained in terms of the logarithmic integrals [10,12]

$$
\int_0^1 d\xi\, \ln^n \xi\, \frac{d}{d\xi} \frac{d\hat{\sigma}_L}{dx}(x, \xi). \tag{6}
$$

The sum of the series, defined by a principal value prescription for the Borel integral, equals

$$
\begin{aligned}
\frac{d\hat{\sigma}_L^{(BS)}}{dx} \ =\ & \int_0^1 d\xi\, \Phi(\xi)\, \frac{d}{d\xi} \frac{d\hat{\sigma}_L}{dx}(x, \xi) \\
& + \left[\frac{d\hat{\sigma}_L}{dx}(x, \xi_L) - \frac{d\hat{\sigma}_L}{dx}(x, 0) \right],
\end{aligned} \tag{7}
$$

where $\xi_L < 0$ is the position of the Landau pole in the strong coupling and the function $\Phi(\xi)$ is specified in Eq. (2.25) of [12]. Infrared renormalons correspond to nonanalytic terms in the expansion of $d\hat{\sigma}_L/dx(x, \xi)$ at small ξ

$$
\begin{aligned}
\frac{d\hat{\sigma}_L}{dx}(x, \xi) \ =\ & \frac{d\hat{\sigma}_L}{dx}(x, 0) + f_1(x)\sqrt{\xi} \\
& + f_2(x)\, \xi \ln \xi
\end{aligned} \tag{8}
$$

and are interpreted as indications of nonperturbative power corrections of the form

$$
\frac{d\hat{\sigma}_L}{dx} = \frac{d\hat{\sigma}_L^{\mathrm{pert}}}{dx} - \frac{\mu_{\mathrm{IR}}}{Q} f_1(x) - \frac{\mu_{\mathrm{IR}}^2}{Q^2} f_2(x) - \ldots \tag{9}
$$

Their size can be estimated by the corresponding ambiguity in the summation of the perturbative series, which is of order of the imaginary part (divided by π) of the sum in (7). Note that identifying the x-dependence of the power corrections in (9) with the x-dependence of the IR renormalon ambiguity or, equivalently, the coefficients of nonanalytic terms in (8) is an assumption which can not be justified from first principles. Since IR renormalons in short-distance quantities are related to ultraviolet ambiguities in higher-twist matrix elements, we refer to this assumption as the 'ultraviolet dominance' of higher-twist corrections.

3. Perturbative series for σ_L

In this section we consider perturbative corrections to σ_L, written as

$$\sigma_L = \sigma_0 \frac{\alpha_s}{\pi} \left[1 + \sum_{n=0}^{\infty} d_n \left(-\beta_0 \alpha_s \right)^n \right], \qquad (10)$$

where σ_0 is the Born total $e^+ e^-$ cross section. As mentioned earlier, we approximate the exact higher-order coefficient by its value in the 'large-β_0' limit, where β_0 is restored from the term with the largest power of N_f at each order. This approximation, called 'naive nonabelianization' in [10], reduces to the familiar BLM prescription for $n = 1$. To see how it works, we rewrite the exact α_s^2 correction in (3) as

$$d_1 = 6.17 - 0.7573/(-\beta_0). \qquad (11)$$

With $-\beta_0 = 0.61$ for $N_f = 5$, neglecting the second term gives an accuracy of about 25%. We have calculated the coefficients d_n in higher orders, in the $\overline{\text{MS}}$ scheme. The 'primary' and 'secondary' quark contributions, $d_n^{[p]}$ and $d_n^{[s]}$, respectively, add to d_n as $d_n = d_n^{[p]}/3 + 2d_n^{[s]}/3$. A few lower order results up to order α_s^4 are

$$d_1^{[p]} = 11/2 \quad d_2^{[p]} = 29.8 \quad d_3^{[p]} = 164, \qquad (12)$$

$$d_1^{[s]} = 13/2 \quad d_2^{[s]} = 46.0 \quad d_3^{[s]} = 369. \qquad (13)$$

The sum of these contributions to all orders is conveniently written in terms of 'enhancement factors' relative to the leading order contribution [12] defined by

$$M^{[p,s]}(\alpha_s) = 1 + \sum_{n=0}^{\infty} (-\beta_0 \alpha_s)^n d_n^{[p,s]}, \qquad (14)$$

so that

$$\sigma_L^{(\text{BS})} = \sigma_0 \frac{\alpha_s}{\pi} \left[\frac{1}{3} M^{[p]} + \frac{2}{3} M^{[s]} \right]. \qquad (15)$$

For various values of $\alpha_s(M_Z)$ we get at $Q = M_Z$

$\alpha_s = 0.110:$
$\quad M^{[p]} = 1.59, \quad M^{[s]} = 1.92 \pm 0.05 .$

$\alpha_s = 0.120:$
$\quad M^{[p]} = 1.68, \quad M^{[s]} = 2.08 \pm 0.08 .$

$\alpha_s = 0.130:$
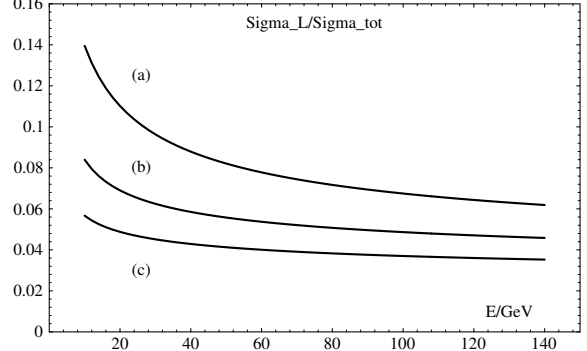$\quad M^{[p]} = 1.79, \quad M^{[s]} = 2.23 \pm 0.12 . \qquad (16)$

Figure 3. Longitudinal fraction in the total $e^+ e^-$ cross section: (c) leading order, (b) next-to-leading order and (a) resummation of all orders in $\beta_0^n \alpha_s^{n+1}$ corrected for the exact $O(\alpha_s^2)$ coefficient.

The given numbers correspond to a principal value definition of the Borel integral and the uncertainties roughly coincide with the size of the minimal term in the series[4]. Let us add the following comments:

(i) The perturbative coefficients in (12), (13) grow rapidly, especially for the secondary quark contribution. This growth is related to an IR renormalon, that indicates a $1/Q^2$ correction to primary quark fragmentation and a $1/Q$ correction to secondary quark fragmentation, see Sect. 4.

(ii) Even though the $1/Q$ power behaviour indicates much larger nonperturbative corrections to σ_L as compared to σ_{tot}, the moderate size of the minimal term of the perturbative series suggests that these corrections are still not large at $Q = M_Z$. The relatively large hadronization correction for σ_L within the JETSET model applied in [2] could thus correspond to higher-order perturbative rather than nonperturbative effects.

(iii) This suggestion is also supported by Fig. 3, where for $\alpha_s(M_Z) = 0.118$ we have plotted the energy dependence of the total longitudinal cross section. Taking into account higher-order perturbative corrections [curve (a)] steepens the energy dependence, such that it is not far from the JETSET prediction, where the steep energy depen-

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[4]The corresponding uncertainty for $M^{[p]}$ is small in comparison with the one for $M^{[s]}$ and is omitted.

dence is due to the hadronization correction. It is worth noting that the parton shower Monte Carlo alone does not yield this energy dependence. Experience with similar calculations suggests that the approximation of resumming only $(\beta_0 \alpha_s)^n$ contributions overestimates radiative corrections, so that we expect a more realistic estimate in between the curves (a) and (b). An exact $O(\alpha_s^3)$ calculation would reduce the theoretical error considerably.

(iv) In [13], universality of the $1/Q$-power correction was assumed and a corresponding unique phenomenological parameter fitted from the difference between the measured average thrust $\langle 1 - T \rangle$ and the theoretical second order prediction. When added to the second order result (3), one obtains a prediction for σ_L consistent with data. There is no conflict between the procedure of [13] and the one presented here, if the phenomenological $1/Q$ correction effectively parameterizes the higher-order perturbative contributions added in our approach. If universality of power corrections holds, these perturbative corrections would also be universal, at least asymptotically in large orders. However, from the point of view presented here, the universality assumption is not required, since higher-order corrections are in principle calculable for each observable.

4. Power corrections

Returning to (8), we quote the expansions for the invariant mass distributions in (5):

$$x \frac{d\hat{\sigma}_L^{[p]}}{dx}(x, \xi) = \qquad (17)$$
$$= \frac{C_F \alpha_s}{2\pi} \left\{ 2x + \xi \ln \xi [8 + 4\delta(1-x)] + \ldots \right\},$$

$$x \frac{d\hat{\sigma}_L^{[s]}}{dx}(x, \xi) = \qquad (18)$$
$$= \frac{C_F \alpha_s}{2\pi} \Big\{ 4(1-x)(2 + 2x - x^2) + 12x \ln x$$
$$+ \quad \frac{4\xi \ln \xi}{5x^2} \left[3 + 30x - 15x^3 + 2x^5 + 15x^2 \ln x \right]$$
$$+ \quad \ldots \Big\}.$$

Interpreting ξ as $(\Lambda/Q)^2$ where Λ is the QCD scale, these expressions are valid for $x > \Lambda/Q$.

We note that for such x, all power corrections are at most of order $1/Q^2$, in agreement with the result from the light-cone expansion of fragmentation processes in [6]. We also see that the power expansion runs in $\Lambda^2/(Q^2 x)$ for the primary quark contribution and $\Lambda^2/(Q^2 x^2)$ for the secondary quark (gluon) contribution. The strong divergence of the second contribution for small x makes it possible for the moments of the x-distribution to have parametrically larger power corrections. Indeed, we find for the two contributions to the total longitudinal cross section

$$\frac{\hat{\sigma}_L^{[p]}}{\sigma_0} = \frac{\alpha_s}{\pi} \left[\frac{1}{3} + 0 \cdot \sqrt{\xi} + 4\xi \ln \xi + \mathcal{O}(\xi^2) \right], \quad (19)$$

$$\frac{\hat{\sigma}_L^{[s]}}{\sigma_0} = \frac{\alpha_s}{\pi} \left[\frac{2}{3} - \frac{5\pi^3}{32} \sqrt{\xi} - 4\xi \ln \xi + \mathcal{O}(\xi^{\frac{3}{2}}) \right], (20)$$

with a $1/Q$ correction for the secondary quark contribution. Assuming ultraviolet dominance of higher-twist corrections, the x-distributions given in (17), (18) can be used to model the x-dependence of power corrections by convoluting the partonic power correction with the leading twist fragmentation function [5,8]. Note that the expressions for the secondary quark contribution differs from the gluon contribution to σ_L in [8], because the series of higher-order fermion loop diagrams does not reduce to the massive gluon calculation performed in [8]. The ensuing additional model dependence in the estimate of higher-twist corrections will be discussed in [5]. Both the calculation here and the calculation with a massive gluon coincide in the essential aspects — power corrections of order $\Lambda^2/(Q^2 x^2)$ for finite x and $1/Q$ for the integrated longitudinal cross section.

In the remainder of this section, we discuss the origin of the $1/Q$ correction in more detail. Adopting for this purpose the massive gluon approximation, one finds that the Borel transform for the gluon fragmentation contribution factorizes as

$$B \left[x \frac{d\hat{\sigma}_L^{[g]}}{dx} \right] (x; u) = x^{-2u} \cdot F(u), \qquad (21)$$

when some terms that can not give rise to a $1/Q$ correction are omitted. Here u is the Borel pa-

rameter and analyticity of $F(u)$ for $|u| < 1$ corresponds to the statement that only $1/Q^2$ corrections arise at finite x. Now, the x-dependence can be absorbed completely into a change of scale in the coupling and the Borel integral is given by

$$x\frac{d\hat{\sigma}_L^{[g]}}{dx} = \int_0^u du\, \exp\left(-\frac{u}{(-\beta_0\alpha_s(xQ))}\right) F(u). \quad (22)$$

The Borel integral (leaving renormalon poles at finite u aside) does not exist for $x < \Lambda/Q$, because it diverges at infinity. This is a manifestation of the fact that for such small x the power expansion breaks down and that power corrections to integrated distributions depend sensitively on how the small-x region is weighted. Indeed, because of the factorization of the x-dependence, the x-integration is trivial and we get

$$B\left[\int_0^1 dx\, x^{1+\gamma}\frac{d\hat{\sigma}_L^{[g]}}{dx}\right] = \frac{F(u)}{1+\gamma-2u}. \quad (23)$$

For the total longitudinal cross section, $\gamma = 0$, and the newly generated pole at $u = 1/2$ corresponds to the $1/Q$ correction discussed before. Note that effects due to color coherence and angular ordering are expected to change the small-x asymptotic behaviour, which could potentially shift the pole to a different value. Clarifying the impact of resummation of small-x logarithms requires similar efforts to those that have been undertaken to understand the effect of Sudakov resummation on power corrections in Drell-Yan production.

Note that the non-uniformity of the power expansion before integration over x does not occur for deep inelastic scattering (DIS) processes. Given the correspondence of IR renormalons with power ultraviolet divergences of higher-twist operators, the difference between fragmentation and DIS must be sought in the renormalization properties of multi-parton correlation functions that appear in the light-cone expansion of [6]. For the longitudinal structure function in DIS, we find that the quadratic power divergence at one-loop of the multi-parton operator

$$g\bar{\psi}(x)\tilde{G}_{\alpha\beta}(vx)x^\alpha\gamma^\beta\gamma_5\psi(-x) \quad (24)$$

takes the form ($\bar{\alpha} \equiv 1 - \alpha$)

$$-\frac{C_F\alpha_s}{4\pi}\frac{\Lambda_{UV}^2}{Q^2}\int_0^1 d\alpha\,(2-\alpha)\Big\{\bar{\psi}(x)\,\slashed{x}\psi(x[\alpha v - \bar{\alpha}])$$
$$+\bar{\psi}(x[\alpha v + \bar{\alpha}])\,\slashed{x}\psi(-x)\Big\}, \quad (25)$$

that is, the form of a convolution with the leading twist contribution. The important point to notice is that the operator spreads only a finite distance on the light-cone under renormalization. In contrast, the multi-parton correlations that appear in fragmentation spread over the entire light-cone under renormalization. When the energy fraction x approaches zero, the operator becomes sensitive to very large longitudinal distances and to how fast the gauge fields decrease at infinity. It is this sensitivity to the behaviour at infinity that causes a $1/Q$ correction in the longitudinal cross section upon integration over x. We will return to this point in detail in a future publication [5].

Acknowledgements

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